

Liquidity, Business Cycles and Monetary Policy: a Simulation

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Abstract

In Kiyotaki and Moore (2008) the authors develop a model in which differences in the liquidity of distinct assets create a link between asset prices and macroeconomic aggregates. Their goal is to build a work-horse model that incorporates liquidity as the cause for the circulation of money, in a dynamic stochastic general equilibrium (DSGE) environment, close enough to the real business cycle (RBC) framework, and thus, suitable for studying monetary policy. In this article, I am interested in studying how this framework can be used to understand the effects of liquidity and technology shocks in the U.S. economy. I calibrate their model and study the impact of adding up liquidity constraints on its RBC performance. I find that liquidity constraints have a remarkable impact on the volatility of investment and consumption. Moreover, I conclude that, in this context, asset prices' volatility is explained as a natural feature of a monetary economy when hit by liquidity shocks.

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1 Introduction:

In Kiyotaki and Moore (2008) - henceforth, KM - the authors develop a model in which differences in the liquidity of distinct assets create a link between asset prices and macroeconomic aggregates. Their ultimate goal is to build a workhorse model that incorporates liquidity as the central cause for the existence of money, in a Dynamic Stochastic General Equilibrium (DSGE) environment, close enough to the Real Business Cycle (RBC) framework, and thus, suitable for studying monetary policy.

KM differs from the usual monetary models in the sense that money is neither defined in a reduced-form way, as conventionally done in Cash-in-Advance and Money-in-Utility models, nor assumed to result from a search environment, as in the prevailing matching models¹. The problem with the first type of models, is that they impose the usage of money rather than explain the causes that justify its circulation. This is not the case with matching models, in which money arises endogenously. However, this type of models is usually not suitable for doing monetary policy, given that tractability is often lost when too rigid assumptions are relaxed².

In KM instead, money circulation is not imposed (meaning that money, as an asset, plays no particularly different role in the economy) and markets are competitive. However, in this model the economy is subject to liquidity constraints, which ultimately define whether money is used or not.

In the model of KM there are two types of agents - workers and entrepreneurs. Two types of assets are transacted: money (in fixed supply) and capital or equity claims of capital (which supply depends on investment and depreciation). While workers play a passive role in this economy, either con-

¹See Lucas and Stokey (1987) and Christiano et al.(1997) for examples of Cash-in-Advance models. Kiyotaki and Wright (1989) and Kocherlakota (1998) illustrate the nature of matching models.

²Lagos and Wright (2005), Aruoba, Waller and Wright (2009) and Huang and Wright (2008) are good examples of attempts to overcome this problem.

suming or saving their income (in the form of money or equity), entrepreneurs have access to a production technology and are subject to an *iid* investment opportunity shock. The fraction of entrepreneurs hit by the shock, have the opportunity to invest in a technology that transforms one unit of output in t into one unit of capital in $t + 1$. Given that, only a percentage of entrepreneurs face these investment opportunities, there is a need for transferring resources from those who do not have been hit by the shock (savers), to those who have (investors). In practice, investors sell equity claims on the output return, resulting from the newly produced capital, at market price q_t , in order to finance their investment. Crucially, investors are needed to employ the newly produced capital and provide the return on equity to its creditors. However, they are not necessarily able to commit to do so. Hence, this limited commitment constraint implies that investors can only pledge a given fraction $\theta \in (0, 1)$ of the newly issued capital. This constitutes a borrowing constraint, in the sense that investors cannot completely leverage the investment opportunity. In other words, funds are not fully transferred from savers to investors and these face the need to finance the investment with their own funds. At this point, liquidity plays its decisive role: in fact, while money holdings are fully convertible into output goods at market price p_t , equity (either in the form of own capital stock or equity claims on others' capital stock) is only convertible into output goods up to a fraction $\varphi_t \in (0, 1)$, at market price q_t , each period.

Whether money plays a role in this economy is defined by how stringent θ and φ_t are. The reason for this lies on the fact that entrepreneurs face a trade-off between holding money and equity: money pays a low return but is completely liquid, whereas equity pays a high return but is only partially liquid. If θ and φ_t are sufficiently low, money circulates and plays the role of providing liquidity to investors. However, if the liquidity constraints are not

significant (θ and φ_t are close enough to one), then the economy is sufficiently liquid and there is no need to hold the lowest return asset (money). Hence, in this context money serves the sole purpose of lubricating the economy. If such purpose is not needed (the economy is already sufficiently lubricated) then money circulation is ruled out. Instead, if liquidity is scarce, money arises endogenously to lubricate the economy and facilitate the flow of funds between savers and investors.

Notice that, even though the authors successfully endogenize money, they do so by creating a reduced-form structure for the liquidity constraints present in the model. In fact, the resaleability constraint φ_t - similarly to the cash-in-advance constraint or the money-in-utility formulation - lacks structural robustness. More concretely, note that φ_t is defined as a Markov stochastic process, so that shocks to the resaleability of equity are present in the model. This is an approach to track the sudden liquidity shortage that has characterized the recent financial turmoil, but we can see right away that it is a poor one. In reality, the formulation of φ_t , as defined in KM, stipulates that the first fraction of old equity holdings is sold at no cost, whereas from the $(\varphi_t)^{th}$ fraction onwards, the investor bears an infinite transaction cost of selling equity. Clearly this is an unreasonable cost structure, which lacks microeconomic fundament³. It is, however, beyond the scope of KM to justify the microeconomic foundations of φ_t , instead it focuses on understanding the effects of liquidity shocks.

Notwithstanding the above, this innovative specification of money and liquidity in a DSGE framework, not only successfully justifies seemingly paradoxical macroeconomic facts, as natural features of a monetary economy that is subject to liquidity shocks, but also allows for the performance of monetary

³This precise fact is referred by John Moore in Claredon Lectures 2 - Liquidity, business cycles and monetary policy (2001). Kiyotaki and Moore (2005) studies in detail an adverse selection justification for φ_t .

policy in a tractable environment⁴.

The RBC literature is nowadays particularly interested in exploring the capacity of state-of-art models, to produce real business cycle statistics that are in line with the ones observed in the data. The better their performance, the more confident one can be when using these models for monetary policy analysis. New-keynesian models like the ones presented in Christiano, Martin Eichenbaum, and Charles L. Evans (2005) or in Frank Smets and Raf Wouters (2007), are equipped to produce time series that can remarkably mimic the data. In fact, these models are so effective in fitting the data that, as I write this article, the European Central Bank is using a version of Smets and Wouters (2007) to inform its monetary policies. Nonetheless, it is not consensual among macroeconomists whether we should support or discourage the usage of New-keynesian models for monetary policy purposes. In fact, a great number of economists fiercely believe that New-keynesian models are not yet useful for policy analysis, as defended by V.V. Chari, Patrick J. Kehoe and Ellen R. McGrattan (2008). The main critique being faced by New-keynesian economists is that they include too many dubiously structural parameters and reduced-form shocks, that are manifestly inconsistent with microeconomic evidence. Neoclassical economists, instead, refuse to include such short-cuts in their models, for the sole purpose of improving their RBC performance, on the grounds that these constitute a severe incongruence with the data, which cannot be ignored when doing policy analysis. From a Neoclassical perspective, as long as this incongruence prevails, New-keynesian models should not be considered reliable for policy analysis, and thus should not be used by central authorities to quantitatively inform their policy-making. Until then, qualitative rules-of-thumb should instead be used.

The model proposed by KM is, in its essence, a congregation of differ-

⁴These paradoxes include the low risk-free rate, the low rate of participation in asset markets and the excess volatility of asset prices.

ent streams of thought and economic theories. Its structural core is a basic Neoclassical model, perfectly competitive and absent of nominal frictions. In this sense this model, avoids the usual criticisms faced by New-keynesian proponents, in what concerns the profligacy of dubiously structural shocks and reduced-form structures that are poorly microfounded⁵. In fact, the model of KM includes only one typically New-keynesian reduced-form shock - the liquidity shock φ_t - as a short-cut for modelling the resaleability constraint presented in Kiyotaki and Moore (2005). As for the investment decision, it embodies the Tobin's q theory, to the extent that investors have an incentive to invest as long as the market price of equity is higher than its cost ($q_t > 1$). Finally, this model proposes an innovative way of including money in a DSGE framework, one that focus on the role of money in lubricating the economy, as the sole cause for its circulation.

Considering the above, despite the reduced-form structure for φ_t , the model of KM is essentially a Neoclassical model, with an original modelling of money and liquidity. My goal is to study to what extent it can contribute to the RBC literature. More concretely, my ambition is to discernibly understand how does each feature of the model in KM contribute to changing the RBC performance of a standard Neoclassical model. In order to do so, I simulate a sequence of models that are simpler versions of KM, calibrated for the U.S. economy. In practice, I start by simulating the simplest version of the model in KM: a standard Neoclassical model, very close to the one presented in King and Rebelo (1999) - also referred to as KR in what follows. Then, I proceed step-by-step, adding up new features to this basic framework, at each step analyzing the RBC performance of the model, until I end up with the economy of KM. My goal is to scrutinize the impact of each new ingredient added, on the RBC behavior of the model. As we shall see, liquidity constraints have a

⁵Check Chari, Kehoe and McGrattan (2008) for a critical view of how suitable New-keynesian models are to do policy analysis.

remarkable impact on the ability of this model to fit the RBC statistics observed for the U.S. economy. In particular, I show that liquidity constraints have a very strong impact on the volatility of investment and consumption, not so much directly, through their impact on the entrepreneurs' behavior, but rather indirectly, for the simple fact that they discourage workers from participating in the asset market. Moreover, the model of KM justifies the paradox of asset prices' volatility as natural feature of a monetary economy that is hit by liquidity shocks φ_t .

The structure of the rest of this article is the following: in Section 2 I present, in a summarized way, the model of KM; Section 3 exposes the above referred Neoclassical de-construction of KM, together with the simulation results of each simplified model; in Section 4 I explain in detail the calibration procedure for the original KM model; Sections 5 and 6 include a detailed study of the shock dynamics in KM and the simulation results for the original KM model; finally in Section 7 I present some concluding remarks.

2 The Model:

In the model of KM there are two types of agents - workers and entrepreneurs - with distinct preferences, although they share the same rate of time preference. Entrepreneurs are subject to investment opportunities that allow them to produce new capital out of consumption goods. Investment opportunities are independent and identically distributed across entrepreneurs and have a constant arrival rate $\pi \in (0, 1)$. Therefore, at each point in time there exists a fraction π of entrepreneurs who have an investment opportunity (investors) and a fraction $1 - \pi$ who do not (savers). Workers, instead play a passive role in this economy. They cannot invest as they are assumed not to face investment opportunities. Hence, they work and either save or consume their income.

There are three types of assets traded, namely a non-durable consumption good (output) Y , equity N and money M . Money is different from equity in the sense that it is completely liquid, meaning that it can be transformed into consumption goods at price p_t with no restrictions (notice that p_t is hereby defined as the inverse of the usual price level). On the contrary, equity is the illiquid asset in this environment, in the sense that it can only be partially transformed into consumption goods at price q_t . The aggregate amount of money in this economy is set fixed at some quantity M .

I proceed by describing this model for each type of agent.

2.1 Workers:

There is a unit measure of workers each with present value utility, at date t , given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t U \left[c_t^w - \frac{\omega}{1+v} (l_t^w)^{1+v} \right] \quad (1)$$

where c_t^w is the consumption level of a typical worker in period t and l_t^w is his labour supply in hours. Assume that $U[\cdot]$ is increasing and strictly concave. Workers do not have investment opportunities, which means that their income relies on the return from their savings and on the wage they get for each hour worked. Money pays no return, whereas equity provides gross profit rate r_t and depreciates at rate $1 - \lambda$ with $\lambda \in (0, 1)$. Hence the flow of funds of a worker can be described as:

$$c_t^w + q_t (n_{t+1}^w - \lambda n_t^w) + p_t (m_{t+1}^w - m_t^w) = w_t l_t^w + r_t n_t^w \quad (2)$$

where m_t^w and n_t^w are holdings of money and equity respectively and w_t is the real wage rate.

The first order conditions of workers include:

$$l_t^w = \left(\frac{w_t}{\omega} \right)^{\frac{1}{v}} \quad (3)$$

This is obtained by equating the marginal rate of substitution between consumption and leisure to the real wage.

Given that there exists a unit measure of workers, the resulting aggregate labour supply is given by:

$$L_t^w = \left(\frac{w_t}{\omega} \right)^{\frac{1}{v}} \quad (4)$$

Later it will be proved that, in a neighborhood of the steady state, workers decide not to save in any kind of asset and thus set their consumption equal to their entire labour income. From this point onwards I assume this is the case:

Claim 1: In a neighborhood of the steady state, workers decide not to save any of their income, thus consuming all earnings resulting from the hours worked.

$$c_t^w = w_t l_t^w$$

2.2 Entrepreneurs:

As with workers, there is a unit measure of entrepreneurs with preferences given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t) \quad (5)$$

Each entrepreneur has access to a constant returns to scale (CRS) technology that produces consumption goods out of capital and labour inputs, according to a Cobb-Douglas:

$$y_t = A_t(k_t)^\gamma(l_t)^{1-\gamma}, \quad (6)$$

where k_t and l_t are respectively the capital and labour used for production.

The capital share of output is denoted by $\gamma \in (0, 1)$. The output produced by entrepreneurs can both be consumed or saved in either equity or money.

Entrepreneurs employ workers in a perfectly competitive labour market, which means that one can define the gross profit rate r_t as the marginal productivity of capital:

$$y_t - w_t l_t = r_t k_t \quad (7)$$

Given k_t , entrepreneurs maximize their gross profit net of labour costs $A_t (k_t)^\gamma (l_t)^{1-\gamma} - w_t l_t$ with respect to l_t . This gives rise to the aggregate labour demand:

$$L_t = K_t \left[\frac{(1-\gamma) A_t}{w_t} \right]^\frac{1}{\gamma} \quad (8)$$

where K_t is the aggregate capital stock. The labour market clearing condition is the one that equates L_t to L_t^w , from (4) and (8). This implies that the equilibrium real wage rate is:

$$w_t = \left(K_t [(1-\gamma) A_t]^\frac{1}{\gamma} \omega^\frac{1}{v} \right)^{\left(\frac{\gamma v}{\gamma+v} \right)} \quad (9)$$

Once we substitute the equilibrium wage rate above into the expression of the gross profit net of labor costs $A_t (K_t)^\gamma (L_t)^{1-\gamma} - w_t L_t$, we can define this to be equal to $r_t K_t$ where r_t is:

$$r_t = a_t (K_t)^{\alpha-1} \quad (10)$$

where:

$$\begin{aligned} a_t &= \gamma \left(\frac{1-\gamma}{\omega} \right)^{\frac{1-\gamma}{\gamma+v}} (A_t)^{\frac{1+v}{\gamma+v}}, \\ \alpha &= \gamma \left(\frac{1+v}{\gamma+v} \right), \end{aligned} \quad (11)$$

The gross profit rate r_t is the marginal productivity of capital. Therefore

it decreases with K_t . Note however that aggregate output, net of labour costs, $Y_t = r_t K_t = a_t (K_t)^\alpha$ is an increasing function of the capital stock. It is also worth emphasizing that, although on the individual level entrepreneurs have access to a constant returns to scale technology, on aggregate, this economy has a decreasing returns to scale production technology. As for the productivity shock A_t it has a positive impact on r_t through a_t .

As previously referred, investment opportunities appear to entrepreneurs with an arrival rate $\pi \in (0, 1)$. These allow them to transform one unit of output in period t into one unit of capital in period $t + 1$. Entrepreneurs finance investment by issuing equity claims on the output produced using the new capital. They sell these claims for q_t units of output and pay one unit of output for each claim. Hence, entrepreneurs decide to invest as long as $q_t > 1$. Conversely, if $q_t < 1$ they prefer not to invest, whereas if $q_t = 1$ entrepreneurs are indifferent between investing and consuming. In this sense, this model embodies the Tobin's q theory of investment.

The equity claims pay r_{t+1} units of output at $t + 1$, λr_{t+2} at $t + 2$, $\lambda^2 r_{t+3}$ at $t + 3$ and so on, for each unit of investment. Investors are not necessarily able to commit to provide these returns on equity to creditors. Due to this limited commitment constraint, investors are only able to pledge a certain proportion of their investment. This is called the Borrowing Constraint and formally states that, investors can only pledge a fraction $\theta \in (0, 1)$ of the newly issued equity i_t . Since the price of the newly issued equity is q_t , they are able to borrow θq_t per unit of investment, which means that for each unit of investment, the downpayment required is $1 - \theta q_t$.

Investors need to use their own funds to finance the downpayment. They can use the money they hold at zero cost. However, they are only able to resell a fraction $\varphi_t \in (0, 1)$, of their old equity holdings (where φ_t is a random variable). This is called the Resaleability Constraint and constitutes the source

of liquidity shocks in this economy.

Notice that, since the own capital stock and equity from other entrepreneurs pay the same return, these assets are perfect substitutes and thus all types of equity can be treated as a single asset. In general an entrepreneur has the following balance sheet:

Balance Sheet at the end of period t	
money: $p_t m_{t+1}$	own equity issued: $q_t n_{t+1}^i$
equity of others: $q_t n_{t+1}^o$	
own capital stock: $q_t k_{t+1}$	Net worth

The net equity of an entrepreneur is:

$$n_{t+1} = n_{t+1}^o + k_{t+1} - n_{t+1}^i$$

At this point the liquidity constraint can be formally written in terms of equity holdings n_t and investment i_t :

$$n_{t+1} \geq (1 - \theta) i_t + (1 - \varphi_t) \lambda n_t \quad (12)$$

An entrepreneur who invests i_t units of output, can only pledge up to a fraction θ of her newly issued equity and resell a fraction φ_t of her old equity holdings λn_t . Therefore, an amount $(1 - \theta) i_t$ of unpledgeable new equity and a quantity $(1 - \varphi_t) \lambda n_t$ of old equity holdings necessarily remain within her balance-sheet.

In addition, money cannot be held short:

$$m_{t+1} \geq 0 \quad (13)$$

The accumulation equation for capital is:

$$k_{t+1} = i_t + \lambda k_t \quad (14)$$

Next, I proceed by analyzing the behavior of investors and savers separately.

2.2.1 Investors

Consider the problem of an entrepreneur who faces an investment opportunity. Her flow of funds is given by:

$$c_t^i + i_t + q_t (n_{t+1}^i - i_t - \lambda n_t) + p_t (m_{t+1}^i - m_t) = r_t n_t \quad (15)$$

where I use the superscript i to designate the investors' variables. The LHS of (15) includes expenditures in consumption, investment and net purchases of equity and money and the RHS includes the investor's income net of labour costs.

Hereafter, in line with KM, I assume that the following claim holds:

Claim 2: Consider the following assumption:

Assumption 1 :

$$\begin{aligned} \Phi(\theta, \varphi_t) = & \pi \lambda \beta^2 (1 - \pi) (1 - \varphi) [(1 - \lambda) (1 - \pi) - (1 - \lambda) \theta - \pi \lambda \varphi] \\ & + [(\beta - \lambda) (1 - \pi) - (1 - \lambda) \theta - \pi \lambda \varphi] [1 - \lambda + \pi \lambda - (1 - \lambda) \theta - \pi \lambda \varphi] \\ & \times [\lambda (1 - \beta) (1 - \pi) + (1 - \lambda) \theta + \lambda (\beta + \pi - \pi \beta) \varphi] > 0 \end{aligned}$$

and assume that it holds. Then, in the neighborhood of the steady state, one can ascertain that:

- Capital is priced above cost ($q_t > 1$);
- Money has strictly positive value ($p_t > 0$);

- Investors want to sell all their money holdings ($m_{t+1}^i = 0$)⁶.

This equilibrium is called a Monetary Economy, in the sense that money has strictly positive value $p_t > 0$. Under the conditions of Claim 2 both liquidity constraints are binding. Given that $q_t > 1$, investors want to liquidate as much asset holdings as they possibly can. This means that we can substitute for i_t , from (12) holding with equality, in the flow of funds of investors, (15), and obtain:

$$c_t^i + q_t^R n_{t+1}^i = \{r_t + [q_t^R (1 - \varphi_t) + q_t \varphi_t] \lambda\} n_t + p_t m_t \quad (16)$$

where $q_t^R = \frac{1-\theta q_t}{1-\theta}$ is the effective replacement cost of equity⁷. The LHS represents the expenditure in consumption and equity, whereas the RHS exhibits the investor's net worth: $\{r_t + [q_t^R (1 - \varphi_t) + q_t \varphi_t] \lambda\} n_t$ and $p_t m_t$ are the value of equity and money respectively. Notice that investors value a fraction φ_t of their depreciated equity at market cost, although the rest of it is valued at replacement cost. This happens because, except when they turn to the market to sell a fraction φ_t of λn_t , investors play the role of equity issuers and value it accordingly.

Investors choose their consumption level and equity holdings by maximizing utility, (5), with respect to c_{t+1}^i and n_{t+1}^i , subject to the budget constraint (16). This already internalizes the conditions for i_t and m_{t+1}^i that result from the liquidity constraints, (12) and (13), verified in equality. The lagrangean

⁶ A sketch of the proof of Claim 2 is provided in Kiyotaki and Moore (2001).

⁷ An investor who decides to invest in one extra unit of equity must make a downpayment of $1 - \theta q_t$. However, with this payment she can only obtain a fraction $1 - \theta$ of the extra equity unit. Hence, the effective price paid by an investor for an extra unit of equity, is given by $q_t^R = \frac{1-\theta q_t}{1-\theta}$.

is:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \log(c_t^i) + \sum_{t=0}^{\infty} E_0 \mu_t \{r_t n_t + [q_t^R (1 - \varphi_t) + q_t \varphi_t] \lambda n_t + p_t m_t - c_t^i - q_t^R n_{t+1}^i\} \quad (17)$$

The marginal conditions of this problem result in the Euler equation:

$$\beta \left\{ \frac{r_{t+1} + [q_{t+1}^R (1 - \varphi_{t+1}) + q_{t+1} \varphi_{t+1}]}{q_t^R} \right\} = \frac{c_{t+1}^i}{c_t^i} \quad (18)$$

Given that preferences are logarithmic, one can derive the optimized level of consumption for each period, as a fraction $1 - \beta$ of the investor's net worth⁸:

$$c_t^i = (1 - \beta) \{r_t n_t + [q_t \varphi_t + (1 - \varphi_t) q_t^R] \lambda n_t + p_t m_t\} \quad (19)$$

The level of investment can be obtained using the flow of funds, (15) and the liquidity constraints with equality, (12):

$$i_t = \frac{1}{1 - \theta q_t} [(r_t + q_t \varphi_t \lambda) n_t + p_t m_t - c_t^i] \quad (20)$$

⁸This claim can be proved as follows: first, I substitute the expression for consumption, (19), in the marginal condition, (18):

$$\beta \frac{r_{t+1} + \psi_{t+1} \lambda}{q_t^R} = \frac{(r_{t+1} + \psi_{t+1} \lambda) n_{t+1}^i + p_{t+1} m_{t+1}^i}{(r_t + \psi_t \lambda) n_t + p_t m_t}$$

where $\psi_t = [q_t^R (1 - \varphi_t) + q_t \varphi_t]$. We know from Claim 2 that $m_{t+1}^i = 0$. Hence:

$$\beta \frac{(r_t + \psi_t \lambda) n_t + p_t m_t}{q_t^R} = n_{t+1}^i$$

Then I substitute the above in the flow of funds, (16), and retrieve the expression for the equilibrium consumption, (19), proving our claim:

$$c_t^i + q_t^R \beta \frac{(r_t + \psi_t \lambda) n_t + p_t m_t}{q_t^R} = (r_t + \psi_t \lambda) n_t + p_t m_t$$

$$c_t^i = (1 - \beta) [(r_t + \psi_t \lambda) n_t + p_t m_t]$$

Condition (20) describes the equilibrium level of investment of the representative investor. In words, it simply states that, in real terms, investment equals the liquid part of net worth that is not used for consumption, once divided by the required downpayment per unit of investment.

2.2.2 Savers:

We turn to savers now. Their flow of funds is much simpler given that they must have $i_t = 0$:

$$\begin{aligned} c_t^s + q_t (n_{t+1}^s - \lambda n_t) + p_t (m_{t+1}^s - m_t) &= r_t n_t, \text{ or} \\ c_t^s + q_t n_{t+1}^s + p_t m_{t+1}^s &= (r_t + q_t \lambda) n_t + p_t m_t \end{aligned} \quad (21)$$

The LHS of (21) includes all gross expenditures of a representative saver, while the RHS contains her net worth. Like investors, savers have logarithmic utility, which means that each saver consumes a fraction $(1 - \beta)$ of her net worth:

$$c_t^s = (1 - \beta) [(r_t + q_t \lambda) n_t + p_t m_t] \quad (22)$$

The following intertemporal marginal conditions for the decision of equity and money holdings, respectively, must hold:

$$\begin{aligned} \frac{1}{c_t^s} &= \beta E_t \left[\pi \frac{r_{t+1} + [q_{t+1} \varphi_{t+1} + (1 - \varphi_{t+1}) q_{t+1}^R] \lambda}{q_t} u' (c_{t+1}^{si}) \right. \\ &\quad \left. + (1 - \pi) \frac{r_{t+1} + q_{t+1} \lambda}{q_t} u' (c_{t+1}^{ss}) \right] \\ \frac{1}{c_t^s} &= \beta E_t \left[\frac{p_{t+1}}{p_t} (\pi u' (c_{t+1}^{si}) + (1 - \pi) u' (c_{t+1}^{ss})) \right] \end{aligned} \quad (23)$$

Above I equate the benefit of consuming one extra unit in period t , with the benefit of saving one unit of the consumption good in the form of $\frac{1}{q_t}$ or $\frac{1}{p_t}$ units of equity or money, respectively, until period $t + 1$. Denote c_{t+1}^{si} (c_{t+1}^{ss}) as the consumption decision in period $t + 1$ of a saver that becomes an investor (remains a saver). Note that in the first equation I must distinguish between

the return on equity of a saver who is to become an investor from the one who is to remain a saver. This results from the fact that savers value equity differently from investors: in fact, investors produce equity and value it at replacement cost, whereas savers are equity buyers and thus value it at market cost. This difference in the return of equity constitutes an idiosyncratic risk faced by entrepreneurs, who at each point in time, are subject to a stochastic shock that defines the type of entrepreneur that they are. There is no idiosyncratic risk, however, in holding money, given that it is totally liquid and therefore pays the same return whether a saver becomes an investor or stays a saver.

The conditions (23), above imply that the following arbitrage condition holds:

$$\begin{aligned} & (1 - \pi) E_t \left[\left(\frac{r_{t+1} + q_{t+1}\lambda}{q_t} - \frac{p_{t+1}}{p_t} \right) \frac{1}{c_{t+1}^{ss}} \right] \\ &= \pi E_t \left[\left(\frac{p_{t+1}}{p_t} - \frac{r_{t+1} + [q_{t+1}\varphi_{t+1} + (1 - \varphi_{t+1}) q_{t+1}^R] \lambda}{q_t} \right) \frac{1}{c_{t+1}^{si}} \right] \end{aligned} \quad (24)$$

Condition (24) defines the liquidity premium in the economy. Note that the LHS includes the expected gain in holding equity instead of money in the event of not having an investment opportunity. Instead, the RHS constitutes the expected gain of holding the most liquid asset (money) instead of the illiquid one (equity), if the entrepreneur turns out to be an investor. Put in other words, this equation prescribes how much should the compensation be to both a saver holding equity instead of money and an investor holding money instead of equity.

2.3 Equilibrium:

The conditions for an equilibrium can be summarized by the following:

$$I_t = \frac{\pi}{1 - \theta_{q_t}} [\beta [(r_t + q_t \varphi_t \lambda) K_t + p_t M] - (1 - \beta) (1 - \varphi_t) \lambda q_t^R K_t] \quad (25)$$

$$\begin{aligned} Y_t^e &= r_t K_t = a_t K_t^\alpha \\ &= I_t + (1 - \beta) [(r_t + q_t \lambda) K_t + p_t M + \pi (1 - \varphi_t) (q_t^R - q_t) \lambda K_t] \end{aligned} \quad (26)$$

$$\begin{aligned} &(1 - \pi) E_t \left[\frac{\frac{r_{t+1} + q_{t+1} \lambda}{q_t} - \frac{p_{t+1}}{p_t}}{(r_{t+1} + q_{t+1} \lambda) N_{t+1}^s + p_{t+1} M} \right] = \\ &= \pi E_t \left[\frac{\frac{p_{t+1}}{p_t} - \frac{r_{t+1} + [q_{t+1} \varphi_{t+1} + (1 - \varphi_{t+1}) q_{t+1}^R] \lambda}{q_t}}{(r_{t+1} + [q_t \varphi_t + (1 - \varphi_t) q_t^R] \lambda) N_{t+1}^s + p_{t+1} M} \right] \end{aligned} \quad (27)$$

where $a_t = \gamma \left(\frac{1 - \gamma}{\omega} \right)^{\frac{1 - \gamma}{\gamma + v}} (A_t)^{\frac{1 + v}{\gamma + v}}$ and $\alpha = \gamma \frac{1 + v}{\gamma + v}$, together with:

$$w_t = \left(K_t [(1 - \gamma) A_t]^{\frac{1}{\gamma}} \omega^{\frac{1}{v}} \right)^{\left(\frac{\gamma v}{\gamma + v} \right)} \quad (28)$$

$$C_t^w = w_t L_t \quad (29)$$

$$Y_t = C_t^w + Y_t^e \quad (30)$$

Above I define Y_t^e and C_t^e respectively as aggregate output net of labour costs and aggregate entrepreneurs' consumption and N_{t+1}^s as the end of period equity holdings of savers.

In what follows I explain how these equilibrium conditions are obtained. The equilibrium consumption and investment conditions for each investor and saver are linear in the beginning of period levels of money and equity holdings. Since there are π investors and $1 - \pi$ savers in this economy, and since the whole stock of capital and money belongs to entrepreneurs, aggregation of

conditions (19) and (22) is straightforward and results in:

$$C_t^i = \pi (1 - \beta) \left\{ [r_t + (q_t \varphi_t + (1 - \varphi_t) q_t^R) \lambda] K_t + p_t M \right\} \quad (31)$$

$$C_t^s = (1 - \pi) (1 - \beta) [(r_t + q_t \lambda) K_t + p_t M] \quad (32)$$

Sum up the above and obtain aggregate consumption of entrepreneurs:

$$C_t^e = (1 - \beta) \left\{ (r_t + q_t \lambda) K_t + p_t M + \pi (1 - \varphi_t) (q_t^R - q_t) \lambda K_t \right\} \quad (33)$$

Using the same method, aggregate the investment condition, (20), and use the aggregate equilibrium consumption of investors, (31), to obtain:

$$I_t = \frac{\pi}{1 - \theta q_t} \left[\beta [(r_t + q_t \varphi_t \lambda) K_t + p_t M] - (1 - \beta) (1 - \varphi_t) \lambda q_t^R K_t \right]$$

The entrepreneurs' goods market clearing condition is:

$$Y_t^e = r_t K_t = C_t^e + I_t$$

Finally, note that, by the end of the period, savers have bought from investors all their money (according to Claim 2) and as much equity as the liquidity constraints allow. This means that they will hold the whole stock of money by the end of the period and an amount of equity equivalent to $N_{t+1}^s = \theta I_t + \varphi_t \pi \lambda K_t + (1 - \pi) \lambda K_t$. Using this fact, aggregate the arbitrage condition (24) and employ it together with the aggregate consumption of investors and savers, respectively (31) and (32), to obtain:

$$\begin{aligned} & (1 - \pi) E_t \left[\frac{\frac{r_{t+1} + q_{t+1} \lambda}{q_t} - \frac{p_{t+1}}{p_t}}{(r_{t+1} + q_{t+1} \lambda) N_{t+1}^s + p_{t+1} M} \right] = \\ & = \pi E_t \left[\frac{\frac{p_{t+1}}{p_t} - \frac{r_{t+1} + [q_{t+1} \varphi_{t+1} + (1 - \varphi_{t+1}) q_{t+1}^R] \lambda}{q_t}}{(r_{t+1} + [q_t \varphi_t + (1 - \varphi_t) q_t^R] \lambda) N_{t+1}^s + p_{t+1} M} \right] \end{aligned}$$

Following the original article by Kiyotaki and Moore, the way to proceed now is to solve for the steady state and to check if claims 1 and 2 are indeed verified. The steady state is characterized by: $A_t = A$, $\varphi_t = \varphi$, $I = (1 - \lambda) K$ and $q_t = q$ and $p_t = p$. In such case the aggregate investment equation, the goods market clearing condition and the arbitrage condition, from (25), (26) and (27) respectively, become:

$$\pi\beta r + \pi\beta v = \left[1 - \lambda + \pi(1 - \beta) \frac{1 - \varphi}{1 - \theta} \lambda\right] - q\theta \left[1 - \lambda + \pi(1 - \beta) \frac{1 - \varphi}{1 - \theta} \lambda\right] - q\pi\beta\varphi\lambda \quad (34)$$

$$\beta r - (1 - \beta)v = \left[1 - \lambda + \pi(1 - \beta) \frac{1 - \varphi}{1 - \theta} \lambda\right] + q(1 - \beta)\lambda \left[1 - \pi \frac{1 - \varphi}{1 - \theta}\right] \quad (35)$$

$$r - q(1 - \lambda) = \pi\lambda \frac{1 - \varphi}{1 - \theta} (q - 1) \frac{\frac{v}{\eta^s} + q}{r + \frac{1 - \varphi}{1 - \theta} \lambda + q \frac{\varphi - \theta}{1 - \theta} \lambda + \frac{v}{\eta^s}} \quad (36)$$

where $\eta^s = \frac{N^s}{K} = \theta(1 - \lambda) + (1 - \pi + \varphi\pi)\lambda$ is the fraction of equity held by savers and $v = \frac{pM}{K}$. One can solve for the first two equations with respect to r and v and obtain its expressions as functions of q . Note that v is closely related to the velocity of money, which is assumed to be constant in the steady state. To see this notice that $v = \frac{pM}{K} = \frac{pM}{Y} \frac{Y}{K} = \frac{1}{V} \frac{Y}{K}$, where I denote V as the velocity of money (recall that, in this context, prices are stated in terms of consumption goods instead of money). Since $\frac{Y}{K}$ is constant in the steady state, it is a necessary condition that V is constant if one wants to find a constant value for v in the steady state.

Compute r and v as functions of q and get:

$$r(q) = \frac{1}{\pi\beta} \left[\begin{array}{c} (1 - \beta + \pi\beta)(1 - \lambda + \pi(1 - \beta)\psi\lambda) \\ -q(1 - \beta)(\theta(1 - \lambda + \pi(1 - \beta)\psi\lambda) + \pi\beta\varphi\lambda - \pi\beta(1 - \pi\psi)\lambda) \end{array} \right] \quad (37)$$

$$v(q) = \frac{1}{\pi} \left[\begin{array}{c} (1 - \pi)(1 - \lambda + \pi(1 - \beta)\psi\lambda) \\ -q(\theta(1 - \lambda + \pi(1 - \beta)\psi\lambda) + \pi\beta\varphi\lambda + \pi(1 - \beta)(1 - \pi\psi)\lambda) \end{array} \right] \quad (38)$$

In order to find the steady state level of q , I simply plug the above into the steady state arbitrage condition, 36, which becomes a quadratic equation for q with a unique positive solution. At this point, it is possible to check that under the conditions of Assumption 1, such solution is guaranteed to be above unity and below $\frac{1}{\theta}$. Indeed this is a Monetary Economy and claims 1 and 2 are verified. It is also possible to prove that near the steady state solution, the following holds: the return on equity of investors is the lowest in this economy, followed by the return on money, which in turn is lower than the return on equity of savers. All these rates of return are lower than the rate of time preference, which is only surpassed by the marginal productivity of capital⁹.

This ordering of interest rates plays a crucial role in this model and is a direct consequence of having a monetary economy resulting from the liquidity constraints. It is interesting to see that close to the steady state solution, not only workers are discouraged from saving, because of too low rates of return, but also savers lose from holding money and equity instead of consuming. It is only because they face the possibility of having an investment opportunity in the future that they decide to save: in such a situation they know that they will need their own funds to finance the downpayment of the investment and allow them to produce the maximum possible amount of capital at a cost lower than the price. It is worth emphasizing that investors want to invest, not because their perceived return on capital is higher than their subjective discount rate, but rather because capital sells at a price higher than its cost ($q_t > 1$).

⁹Check Kiyotaki and Moore (2001) for a sketch of the proof.

Notice that the wedge between the rates of return in this economy follows exclusively from the fact that liquidity constraints are present. If there were no liquidity constraints, funds would flow from savers to investors until this wedge had been shut down and capital price equaled its cost. However, because of this financial friction, this economy will restrain the flow of funds in a situation in which investment is still in shortage, but no more funds are available to invest. Consequently production stops below the First Best level.

The link between asset prices and business cycles lies in the precise fact that shocks to the resaleability constraint have an impact on both the price of equity and the amount of investment downpayment that investors are able to make. If a liquidity shortage occurs (φ_t suffers a negative shock), the economy becomes more constrained and less funds will flow from savers to investors. Investment decreases and pulls capital accumulation downwards. Less capital stock results in a lower real wage rate and in a higher gross profit rate (which is equal to the marginal productivity of capital in this environment). Entrepreneurs will value capital relatively more and consequently its price q_t will rise. Facing lower wages, workers decrease their consumption. As for entrepreneurs, even though they increase their consumption on impact (due to their additional inability to invest), as their net worth decreases in value they will necessarily decrease their consumption level. In the end, investment, capital accumulation, consumption and output will all break down and an economic recession kicks in. Clearly there exists a feedback from the asset market on macroeconomic aggregates, as the authors defend.

3 Neoclassical De-construction:

Above, I have summarized the model presented in KM. I am mainly interested in understanding how this framework can be used to understand the effects of liquidity and technology shocks in the U.S. economy. However, before I

proceed to calibrate and simulate this model, it is crucial to understand how it works. For this purpose, in this section I start by simulating a standard Neoclassical growth model and then proceed step by step, adding new features to this basic framework until I end up with the KM economy. The first step will be to review the simulation results obtained in the model by King and Rebelo (1999), with the difference that I will not include deterministic technology growth in the model. Next, I will use a particular specification of the preferences in KM, i.e. the preferences in Greenwood, Hercowitz, and Huffman (1988) - also referred to as GHH from this point onwards. In a third step towards the KM model, I will introduce a distinction between two types of agents in the economy. Finally, in Section 5, I will include the two financial constraints presented above and allow for liquidity shocks. In each step I will be checking for changes in volatility, comovement and persistence when stochastic shocks are fed into the model and the simulation is performed. My goal is to compare the ability of each model to replicate fundamental real business cycles properties of the U.S. economy and to understand in which way the liquidity constraints may bring about changes in the ability of Neoclassical models to reproduce these RBC properties. In practice I will be deriving some crucial RBC moments and comparing them with the corresponding statistics for the U.S. economy, as carefully presented in Stock and Watson (1998). Table

1 synthesizes these statistics:

Table 1: Business Cycle Statistics for the U.S. Economy				
Variable	St Dev	Relative St Dev	Autocorrelation	Correlation with Y
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
L	1.79	0.99	0.88	0.88
Y/L	1.02	0.56	0.74	0.55
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	-0.35
A	0.98	0.54	0.74	0.78

Values in Table 1 are presented in log-deviations from the steady state and in percentage terms.

3.1 Benchmark Neoclassical model:

In the paper by KR the authors consider a standard Neoclassical model in which households have to choose a path for consumption, labour supply and asset holdings. This problem can be described by:

$$\begin{aligned}
& \underset{\{c_t, l_t, a_{t+1}\}_{t=0}^{\infty}}{Max} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \Lambda \frac{(1 - l_t)^{1-\eta} - 1}{1 - \eta} \right] \\
& s.t. \quad c_t + a_{t+1} - (1 - \delta) a_t = w_t l_t + r_t a_t
\end{aligned}$$

where c_t is the consumption level, a_t is the amount of assets held and l_t stands for labour supply. Households receive a rental rate r_t for the assets held and real wage w_t for the hours worked. These are paid by competitive firms that have access to a Constant Returns to Scale (CRS) technology that uses capital

and labour as inputs. The production function is a Cobb-Douglas:

$$y_t = A_t (k_t)^\gamma (l_t)^{1-\gamma}$$

where l_t is the labour demand, k_t is the stock of capital of each firm and A_t is an aggregate productivity coefficient. Since this is a closed economy environment it results that $a_t \equiv k_t$. This economy is hit by stochastic shocks A_t that follows an $AR(1)$ Markov process $\log A_t = \rho_A \log A_{t-1} + \varepsilon_A$, where ε_A is an *iid* random variable with standard deviation σ_A . Finally, contrary to the case of KR, the model presented here is absent of growth. I perform this modification to the KR framework given that there is also no economic deterministic growth going on in the KM framework. This very simple model is then calibrated in such a way that some stylized facts for the U.S. economy are met:

KR Model Calibration:						
β	Λ	γ	δ	η	ρ_A	σ_A
0.984	3.48	0.333	0.025	1	0.979	0.0072

As in KR, I compute the RBC moments for this economy after simulating

the model. These are presented in Table 2.

Table 2: Business Cycle Statistics for King and Rebelo RBC model				
Variable	St Dev	Relative St Dev	Autocorrelation	Correlation with Y
Y	1.37	1.00	0.72	1.00
C	0.61	0.45	0.78	0.95
I	4.50	3.29	0.71	0.99
L	0.64	0.47	0.71	0.97
Y/L	0.75	0.55	0.76	0.98
w	0.75	0.55	0.76	0.98
r	0.05	0.04	0.71	0.96
A	0.93	0.68	0.72	0.99

The simulated shocks produce a model that is almost as volatile as the data for the U.S. economy. In relative terms, consumption is two thirds smoother than output and investment three times as volatile. Although consumption lacks some volatility these results are more or less in line with the data. The model fails though, in explaining the observed volatility in hours: the simulated standard deviation of 0.64% compares to the observed 1.79% in the U.S. data. Conversely, the real interest rate observed volatility of 0.30% is underestimated by the model which leaves this standard deviation at 0.05%. Persistence is reasonably well predicted by the model. Nevertheless, underestimation is present in most variables and is particularly relevant for consumption and output. On the contrary, both real prices are slightly more persistent in the model than in the data. Finally, comovement comes with the right sign for all variables but the real interest rate. Indeed, in Table 1 one can see that the real interest rate is countercyclical in the U.S. economy, whereas in Table 2 the simulated correlation coefficient between the real interest rate and output

is positive and close to one¹⁰.

Moreover, in general, variables are too correlated with output when compared with the data, particularly real wages, for which the observed correlation coefficient is 0.12 whereas the model predicts it to be of the order of 0.98.

Despite the referred flaws, I use this model as a benchmark for a good RBC simulation and compare it with the next simulations.

3.2 Neoclassical model with GHH preferences:

Consider now a slightly different Neoclassical Growth model where there is a unit measure of infinitely lived consumers with preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t U \left[c_t - \frac{\omega}{1+v} (l_t)^{1+v} \right]$$

where $U(\cdot)$ is increasing and strictly concave. Note that this formulation for preferences is the one present in KM. Following GHH, let the functional form for the utility function be:

$$U(c_t, l_t) = \frac{\left(c_t - \frac{\omega}{1+v} (l_t)^{1+v} \right)^{1-\chi} - 1}{1-\chi}$$

From this type of preferences one can derive the marginal utility of consumption $U_c(t) = \frac{1}{\left(c_t - \frac{\omega}{1+v} (l_t)^{1+v} \right)^\chi}$ and the marginal disutility of labour $U_l(t) = \frac{-\omega l_t^v}{\left(c_t - \frac{\omega}{1+v} (l_t)^{1+v} \right)^\chi}$. The equilibrium conditions for households are:

$$l_t = \left(\frac{w_t}{\omega} \right)^{\frac{1}{v}} \tag{39}$$

$$1 = \beta E_t \left[(r_t + 1 - \delta) \left(\frac{c_t - \frac{\omega}{1+v} (l_t)^{1+v}}{c_{t+1} - \frac{\omega}{1+v} (l_{t+1})^{1+v}} \right)^\chi \right] \tag{40}$$

As above, firms equate the marginal utility of capital and labour to its respec-

¹⁰There is some debate, though, on whether the real interest rate is indeed countercyclical. In fact, Teles and Brito (2004) find that the real interest rate can be procyclical, in times of high inflation rate.

tive rates of return:

$$r_t = \gamma A_t \left(\frac{l_t}{k_t} \right)^{1-\gamma} \quad (41)$$

$$w_t = (1 - \gamma) A_t \left(\frac{k_t}{l_t} \right)^\gamma \quad (42)$$

Finally, the goods market clearing condition and the law of capital accumulation are:

$$y_t = A_t (k_t)^\gamma (l_t)^{1-\gamma} = c_t + i_t \quad (43)$$

$$i_t = k_{t+1} - (1 - \delta) k_t \quad (44)$$

The above, together with the law of motion for the technology shock $\log A_t = \rho_A \log A_{t-1} + \varepsilon_A$, constitute the equilibrium conditions for this model.

Next, I proceed as above and calibrate the model so as to replicate some stylized features of the U.S. economy. As in KR, I set $\beta = 0.984$, so that the steady state annual interest rate equals 6.5%. Following the same paper, I choose the depreciation rate to be 10% per annum (which means that I assume $\frac{K}{Y} = 10$ and $\frac{I}{Y} = 0.25$). This leads to a quarterly gross depreciation rate $1 - \delta = 0.975$. For the utility function parameters I set $v = 0.6$ as in GHH, but choose a smaller value for the coefficient of risk aversion $\chi = 0.5$ (rather than 1.001 as in GHH), to compensate for the inexistence of investment shocks in this model, contrary to the case in the GHH article. At last, with this calibration in mind, I choose $\omega = 4.09$, so that the average fraction of time spent working in the steady state equals 20%, as is commonly accepted in the literature for the U.S. economy. The same technology shocks as above are then performed and the resulting RBC moments are presented in Table 3.

Consumption volatility increases at the expense of a lower investment variability: the standard deviation of consumption is now approximately 60% that

of output, while investment sees its variability slightly decreased to 2.84 times the volatility of output, but still roughly in line with the data. A notable improvement is made on the volatility of labour, which now accounts for 63% that of output. One last remark to the volatility of real wages which is now exactly in line with the data. Almost no changes are noticeable in what concerns persistence: as in King and Rebelo, this model still underestimates the persistence of most variables (only real prices presidences are overestimated). Finally, comovement results are also similar to those in KR, even if they are further away from the ones from the data.

Table 3: Business Cycle Statistics for a basic, GHH utility, RBC model				
Variable	St Dev	Relative St Dev	Autocorrelation	Correlation with Y
Y	1.62	1.00	0.73	1.00
C	0.90	0.56	0.77	0.98
I	4.60	2.84	0.72	0.99
L	1.01	0.63	0.73	1.00
Y/L	0.61	0.38	0.73	1.00
w	0.61	0.38	0.73	1.00
r	0.06	0.04	0.72	0.97
A	0.94	0.58	0.72	0.99

One can therefore conclude that, changing the functional form of preferences, does not significantly interfere with the capacity of the Neoclassical framework, to produce time series for the macroeconomic aggregates and prices that perform reasonably well in predicting the observed volatility, persistence and comovement in the U.S. data. Actually, despite the increased overestimation of comovement, GHH preferences equip the Neoclassical model with a better capacity to replicate the volatility in hours and consumption verified in the data.

3.3 Model with workers and capital owners:

In this section I introduce a distinction between two types of agents: workers and capital owners. These play different roles in the economy, similarly to what happens in KM, but without the financial constraints. As in KM, I assume that workers want to consume all their labour income. This is an absurd claim, given that nothing structurally prevents workers from wanting to save and smooth consumption. Indeed, in this framework the rate of return on capital equates the rate of time preference, thus providing an incentive for workers to save. Nonetheless, I keep this assumption, on the grounds that it is a step forward towards the model in KM. In practice, as we have already seen, this claim is verified in a neighborhood of the steady state in the model of KM, provided that the liquidity constraints are stringent enough. In the end, this fact will play a crucial role on the performance of the model to replicate the U.S. real business cycles statistics and therefore is worth analyzing separately, in the absence of liquidity constraints.

Assume there is a unit measure of both workers and capital owners. Workers have preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(c_t^w - \frac{\omega}{1+v} (l_t^w)^{1+v} \right)^{1-\chi} - 1}{1-\chi}$$

as in the previous case, whereas capital owners have preferences of consumption, c_t^k :

$$E_0 \sum_{t=0}^{\infty} \beta^t \log \left(c_t^k \right)$$

and have access to a Cobb-Douglas production technology $y_t = A_t (k_t)^\gamma (l_t^k)^{1-\gamma}$, where l_t^k is the labour demand. With the output of their production, capital owners pay for labour, consume and invest in new capital. The flow of funds is:

$$c_t^k + k_{t+1} - (1 - \delta) k_t = A_t (k_t)^\gamma \left(l_t^k \right)^{1-\gamma} - w_t l_t^k$$

The Lagrangean for the problem of a representative capital-owner is:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \log \left(c_t^k \right) + \sum_{t=0}^{\infty} E_0 \mu_t \left[A_t (k_t)^\gamma \left(l_t^k \right)^{1-\gamma} + (1 - \delta) k_t - w_t l_t^k - c_t^k - k_{t+1} \right]$$

Derive the first order conditions of the Lagrangean and work them out to obtain the equilibrium conditions:

$$\frac{1}{c_t^k} = \beta E_t \frac{1}{c_{t+1}^k} \left[\gamma \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right] \quad (45)$$

$$w_t = (1 - \gamma) A_t \left(\frac{k_t}{l_t^k} \right)^\gamma \quad (46)$$

From the Euler Equation, (45), one can see that the gross profit, which I define as r_t , is equal to its marginal productivity:

$$r_t = \gamma \frac{y_t}{k_t} \quad (47)$$

As in the KM model, preferences are logarithmic and, therefore, the optimal level of consumption for a representative capital owner is a fraction $1 - \beta$ of his net worth. In this context, the net worth is the valuation of the capital held by the agent at any given point in time, for a given rate of return r_t and depreciation rate δ . Hence, consumption results in:

$$c_t^k = (1 - \beta) (r_t + 1 - \delta) k_t \quad (48)$$

As for workers, I set their consumption fixed and equal to the income from

labour:

$$c_t^w = w_t l_t^w \quad (49)$$

The labour supply, instead, results from equating the marginal rate of substitution to the real wage rate:

$$l_t^w = \left(\frac{w_t}{\omega} \right)^{\frac{1}{v}} \quad (50)$$

The aggregate conditions corresponding to expressions (45) to (50), together with the goods market clearing condition $Y_t = C_t^k + C_t^w + I_t$ and the law of motion for capital $I_t = K_{t+1} - (1 - \delta) K_t$, constitute the equilibrium conditions for this economy.

I apply the same calibration to this model as the one in the model without the distinction between workers and capital owners and produce the same technology shocks. The RBC statistics for this model are presented in Table 4. Comparing the simulation moments of this model with the ones presented in Table 3, we can conclude that, intuitively, non-optimizing workers lead to a much greater consumption volatility than before - it now accounts for almost 85% of the output's standard deviation -, investment loses a great amount of variability - it is now only 1.63 times as volatile as output - and all other variables keep approximately the same standard deviation. In what concerns persistence and comovement, this model produces statistics that are identical

to the ones resulting from the previous model.

Table 4: Business Cycle Statistics for a model with workers and capital owners and GHH utility				
Variable	St Dev	Relative St Dev	Autocorrelation	Correlation with Y
Y	1.60	1.00	0.72	1.00
C	1.35	0.84	0.73	1.00
I	2.60	1.63	0.72	0.99
L	1.00	0.63	0.73	1.00
Y/L	0.60	0.38	0.73	1.00
w	0.60	0.38	0.73	1.00
r	0.07	0.04	0.72	0.99
A	0.94	0.59	0.72	0.99

Hence, when one distinguishes between workers and capital owners and forces the first to consume their entire labour income, the model performance in predicting the RBC features of the U.S. economy changes considerably. This results from the fact that consumption and investment volatility, respectively, increase and decrease significantly. This comes at no surprise, given that, by inhibiting workers from saving, I am depriving a proportion of agents from the ability to smooth consumption. In boom times, consumption from workers is too high and funds that would otherwise be used for investment are consumed. Conversely, during recession periods, workers consume too little, given that they do not dispose of any savings to smooth their behavior. This fact will play a crucial role in the simulation of the model by Kiyotaki and Moore, in which case the claim that workers consume their labour income is legitimated by the fact that liquidity constraints indeed prevent workers from wanting to save.

4 Calibration:

Before I take the final step and perform the dynamic analysis and simulation of the model in KM, I must go through the details of the calibration. As before, I am calibrating this economy assuming that each period represents a quarter. First, following KR, assume that the capital-output ratio is $\frac{K}{Y} = 10$ and that the investment-capital ratio is $\frac{I}{Y} = 0.25$. These lead to a quarterly gross rate of depreciation $\lambda = 1 - \frac{I/Y}{K/Y} = 0.975$. In what concerns the elasticity of labour supply, I set $v = 0.6$ as described above. Note that in the a First Best solution, with no financial constraints, the gross profit rate (or the marginal productivity of capital) is equal to the rate of time preference. Such equilibrium is one of the possible outcomes of the current model - one in which the liquidity constraints are set to $\theta = \varphi = 1$ and the gross profit rate r_t corresponds to the real interest rate - hence, I use this result to calibrate the discount factor β : again, following KR set the real interest rate at 6.5% per annum in the First Best scenario, which implies that β is set to 0.984 on a quarterly basis. With this calibration in mind, I choose a value for ω that guarantees that the average fraction of time spent working in the steady state equilibrium equals 20%, as is commonly accepted in the literature for the U.S. economy.

My aim is to calibrate this economy in such a way that the level of q resulting from the steady state equilibrium is in line with the average q for the U.S. economy, as estimated in Laitner and Stolyarov (2003) - LS in what follows. In this article the authors use U.S. annual investment data from 1953 to 2001 in order to estimate a time series for q . According to their study, the level of q averaged 1.2075 during their period of analysis. I consider this to be the steady state level for q and calibrate the financial constraints accordingly.

Consider the resaleability constraint first. It is not straightforward to find an empirical support for this type of financial restriction. However, a detailed estimation of the average post-war liquidity share in the U.S. economy is pre-

sented in Negro, Eggertsson, Ferrero and Kiyotaki (2009) - henceforth, NEFK - and used to match the same resaleability constraint. I follow this article in order to calibrate the stochastic liquidity shock. I construct an historical series from 1952|Q1 to 2008|Q4 for the liquidity share in the U.S. as defined in NEFK: $LS_t = \frac{p_t M_t}{p_t M_t + q_t K_t}$. From this data I extract the average liquidity share and its autocorrelation and standard deviation coefficients. These are given by $\varphi_{ss} = 0.11$, $\rho_\varphi = 0.969$ and $\sigma_\varphi = 0.029$ and provide an estimation for steady state level of φ and for its law of motion, when calibrated for the U.S. economy¹¹. In what concerns the borrowing constraint I choose θ so that the leverage ratio in this economy is in line with the observed average debt-to-equity ratio of four in the U.S. economy - in line with Gertler and Kiyotaki (2009). For each unit of investment, an entrepreneur borrows an amount θq from savers and finances $1 - \theta q$ with her own funds. Hence I set:

$$\begin{aligned} 4 &= \frac{\theta q}{1 - \theta q} \\ \Rightarrow \theta &\approx 66.3\% \end{aligned}$$

I am left with the investment opportunity arrival rate π to calibrate. Since there is no obvious direct evidence for this rate, I choose the value of π in such a way that the level of q resulting from the steady state equilibrium in this economy is given by $q = 1.2075$, in line with LS, as previously referred¹².

Such calibration implies that each quarter an entrepreneur has a probability

¹¹Notice that, although φ_t is defined as the rate at which an entrepreneur can alienate her old equity holdings, in order to fund her investment opportunity (i.e. as a flow variable), hereby I am tracking it with a picture of how liquid an economy is on average (i.e. as a stock variable). Clearly the stock of liquidity in a given economy is intrinsically related with the velocity with which one is converting the illiquid asset (equity) into the liquid one (money). Following NEFK, I argue that, on average, the velocity at which equity is exchanged for money equates the aggregate liquidity share.

¹²In Negro, Eggertsson, Ferrero and Kiyotaki (2009), the authors directly calibrate the investment opportunities arrival rate at 7% per quarter. They argue that this value is probably an upper bound for this coefficient.

Instead in Gertler and Kiyotaki (2009) the authors simply set this rate at 25% so that each year an entrepreneur has on average one investment opportunity.

of approximately 1.82% of facing an investment opportunity.

The calibration of this model is summarized in the following table:

KM Model Calibration:						
β	v	δ	ω	θ	φ	π
0.984	0.6	0.975	4.67	0.663	0.11	0.018

In what concerns the stochastic technology coefficient A_t it is easy to choose a law motion: following KR, simply set $\rho_A = 0.0979$ and $\sigma_A = 0.0072$ as I have already done for the simplified Neoclassical models presented above.

5 Dynamics:

In this section I analyze the behavior of the macroeconomic aggregates and prices in the model by KM, when hit by shocks in the stochastic variables. Although the model of KM incorporates only two stochastic variables (A_t and φ_t), I treat an extra couple of parameters as such, in order to understand how the model reacts to sudden changes in both the arrival rate of investment π and in the borrowing constraint θ .

I start with a shock to the productivity variable A_t and suppose that its law of motion is given by $\log(A_t) = \rho_A \log(A_{t-1}) + \varepsilon_t^A$, where the error term ε_t^A is the *iid* technology shock. I am interested in analyzing the impulse response functions of prices and aggregates to unexpected changes in the technology coefficient. Following the above described calibration, I set $\rho_A = 0.979$ and I consider a 1% shock to A_t . The impulse response functions for this shock are presented in Figures 2a and 2b in appendix.

When the productivity shock hits the economy, the marginal productivity increases and departs further from the rate of time preference. This means that the economy becomes virtually more constrained: each unit of investment funds is now relatively more valuable. Ideally, capital accumulation would

respond and drive its marginal productivity downwards until the subjective discount rate was reached. In this setup, though, capital production is constrained, which means that an increase in the marginal productivity of capital boosts its value and leads to an increase in its price q_t . A higher equity price decreases the downpayment required per unit of investment. Hence, investment increases and pushes capital accumulation upwards. Given that money is an input for investment, the increase in q_t drives the price of money p_t upwards. The upward reaction in both prices and in the productivity of capital, in turn, increases the value of entrepreneurs' net worth, this way fuelling the rise in investment and consumption. In the mean time, a higher capital stock leads to an increase in the real wage rate, which results in a higher consumption level of workers. Entrepreneurs, however, will consume less on impact, due to the increased attractiveness of investment that constitutes a strong substitution effect. On aggregate, consumption starts by responding downwards, only to increase as the shock vanishes and output expands, allowing for the income effect to exceed the substitution effect.

From the above we can conclude that this model includes an amplification effect. Note that, after the liquidity shock, investment jumps in response to the rise in the marginal productivity of capital, as usual. However, as prices rise in response to the higher value of capital, the net worth of entrepreneurs increases in value, providing more funds for investment. Furthermore, also the required downpayment $1 - \theta q_t$ decreases as the price of capital rises, providing an additional source of investment growth.

Consider a persistent negative shock to the resaleability constraint. As defined in Section 4, φ_t follows an $AR(1)$ Markov process that is stationary around $\varphi_{ss} = 0.11$. Thus we have $\varphi_t = (1 - \rho_\varphi) \varphi_{ss} + \rho_\varphi \varphi_{t-1} + \varepsilon_t^\varphi$ where ε_t^φ is the *iid* resaleability shock. Figures 3a and 3b show the effects on the macroeconomic aggregates and prices of an unexpected persistent negative

shock to φ . Following the calibration in Section 4 set $\rho_\varphi = 0.969$. As for the scale of the shock I consider a liquidity deterioration in which φ_t decreases by 1p.p..

When φ_t decreases investors see their ability to raise funds, out of their previously held equity, reduced. The amount of equity converted into investment funds decreases, meaning that investment decelerates and capital accumulation drops, pushing its marginal productivity upwards. In what concerns the price of equity, two contradictory forces are at place. On the one hand, equity is now less desirable, given that is relatively less liquid than before. Entrepreneurs perform a so called *flight to quality* and demand more money and less equity, this way driving the price of equity q_t downwards. On the other hand though, and most importantly, capital is now scarcer and its marginal productivity higher. This means that each unit of equity used by entrepreneurs for investment is now more valuable, and this ultimately pushes the price of equity q_t upwards. We can see from Figure 3b that, in the end, this second effect prevails and q_t raises in response to a negative shock to φ_t . Intuitively, since we are further departing from a situation where there is no resaleability constraint on equity and where the price of capital equals its cost ($q_t = 1$), it befits naturally that a sudden reduction in liquidity drives the price of capital upwards. The higher demand for money resulting from the *flight to quality*, together with the increased value of investment, both lead to an increase in its price p_t . As for consumption, it will be driven upwards on impact, due to the substitution effect created by the increased funding difficulties. However, as capital accumulation freezes and investment slows down, output falls and inevitably consumption is pushed downwards. Note that consumption of each type of agent reacts differently: while entrepreneurs decide to consume more on impact, only to decrease their consumption as output slows down, workers immediately decrease their consumption in response to lower wages.

Interestingly, in the case of a stochastic shock to liquidity, this model implies a absorption rather than an amplification effect. In fact, as the resaleability shock hits the economy, investment breaks and leads capital accumulation, consumption and output downwards. However, as asset prices react positively, entrepreneurs' pain is relieved by an increase in their net worth value, which, in turn, cushions the drop in the macroeconomic aggregates.

It is clear from this analysis that a liquidity shock can qualitatively reproduce an economic recession like the one the world economy has been facing since the summer of 2007. Furthermore, it is interesting to see that even a transitory shock to φ_t can produce a long-lived recession: if we set $\rho_\varphi = 0$ and simulate the same liquidity shock as above, it can be seen from Figure 3c and Figure 3d that, although prices and investment swiftly return to normal as the shock vanishes, capital accumulation, consumption, output, real wages and the gross profit rate take a lot longer to retrieve to their steady state level.

If, instead, we consider a shock to the borrowing constraint coefficient θ , very similar qualitative results are obtained. Assume that, θ_t follows an $AR(1)$ Markov process: $\theta_t = (1 - \rho_\theta) \theta_{ss} + \rho_\theta \theta_{t-1} + \varepsilon_t^\theta$. In Section 4 I have calibrated the steady state borrowing constraint parameter $\theta_{ss} = 0.663$, but not the persistence or scale of its shock, given that θ is set constant in KM. As I am only interested in checking for the qualitative impact of a persistent shock to the borrowing constraint, I take a shortcut and use the same persistence coefficient as I did with the resaleability constraint: $\rho_\theta = 0.969$. The graphics for the impulse response functions of a sudden decrease of one *1p.p.* in θ_t , are depicted in figures 4a and 4b.

When θ_t decreases, a higher downpayment will be required per unit of investment. Investors will, suddenly, be incapable of leveraging as much output claims as they did before. Hence, the ability of investors to issue inside equity is further reduced, this way depriving the economy from an important source

of liquidity. Aggregate investment decreases due to the shortage of funds and along with it capital accumulation decelerates. For lower levels of capital stock, the marginal productivity of capital r_t rises. As with the shock to φ_t , the price of equity suffers two distinct effects: the first one pushes q_t upwards and results from the increased value of capital (income effect), whereas the second one drives q_t downwards and results from the entrepreneurs' portfolio adjustment (substitution effect). In the end the income effect prevails and q_t responds upwards. As before, the price of money p_t is driven upwards by the adjustment in the portfolio composition and the increased value of investment. Finally, like with the shock to φ , aggregate consumption responds positively at first, but eventually starts to decrease as the income effect surpasses the substitution effect.

One can therefore conclude, that a reduction in the pledgeable fraction of investment leads to the same qualitative results produced by a contraction in the resaleability of equity, even though the triggering shock is structurally different: in fact, a shock to θ_t has an impact directly on the downpayment of investment and constitutes a shock to the leveraging ability of investors; a shock to φ_t , instead, decreases the ability of investors to resell their equity, with only indirect effects on the required downpayment.

Finally, consider a persistent shock in the arrival rate of investment opportunities π . The impulse response functions of macroeconomic aggregates and prices, with respect to a *1p.p.* increase in π , are plotted in figures 5a and 5b respectively. Define π_t as an *AR*(1) Markov process that is stationary around some steady state which I denote as π_{ss} . This way one can write $\pi_t = (1 - \rho_\pi) \pi_{ss} + \rho_\pi \pi_{t-1} + \varepsilon_t^\pi$ where ε_t^π is an *iid* shock. Recall that there is no aim at correctly microfounding the arrival rate of investment opportunities. Instead, I choose a calibration procedure that forces this economy to be in a monetary equilibrium, with a steady state Tobin's q that is in line with LS.

For this reason, I pick $\pi_{ss} = 0.018$ and simply set the persistence coefficient to be $\rho_\pi = 0.95$, so that we can capture the effects of a persistent decrease in the investment opportunities arrival rate. I study the impact of a rise in π_t of one percentage point.

In order to understand the qualitative impact of a shock to π_t , it is useful to think about an extreme situation in which entrepreneurs face an investment opportunity with probability one ($\pi = 1$): in such case, investment opportunities are useless, given that, although every entrepreneur wants to sell equity claims, there is no demand for them, for the simple reason that savers have been extinguished. The only logical equilibrium in such situation is to have $q_t = 1$ and $p_t = 0$, which means that investors are indifferent between investing and saving, and money, as a consequence, plays no role in this economy. Therefore, if one considers a positive shock that departs from π_{ss} , it is only reasonable to expect that when the arrival rate of investment opportunities rises, prices of both assets should fall. From the equilibrium conditions, it can be seen that a rise in π_t leads to an increase in the aggregate investment that is independent of the downpayment required per unit of output invested: investment rises simply because there are more people investing. Capital accumulation necessarily soars and, in the mean time, consumption breaks down, given that there is now a larger fraction of agents in the economy who have access to investment and thus value consumption relatively less.

In a sense, a positive shock to π_t increases the ability of this economy to avoid the liquidity constraint. Liquidity is the more important the more infrequent are investment opportunities. The main reason for this to happen hinges on the fact that with a lower fraction of savers in the economy, there are less funds to be transferred from savers to investors and, therefore, the restringency in the flow of these funds (i.e. the liquidity constraint) loses importance and the economy approaches the First Best resource allocation.

6 Simulation:

In this section I simulate the model in KM with two distinct structures. First I assume that only the technology is stochastic and then also consider φ_t to be a stochastic variable and simulate the model with its original structure, as presented in Kiyotaki and Moore (2008). I calibrate the model following Section 4 and simulate it using Dynare (v.4.0.4) - the same software package was used to simulate the above presented models. The results for the RBC moments of these simulations are displayed in Table 5 and Table 6, respectively, for the two cases above.

I start by simulating a version of KM in which only A_t is stochastic. If one analysis the evolution of the RBC statistics in Section 3, as I introduce new ingredients to a standard Neoclassical model, one can clearly see that the crucial changes have occurred when I have distinguished between workers and capital owners, and forced workers to consume their entire income. This distinction, as explained above, brings a lot less volatility to investment and much more variation to consumption. Recall that when I first introduced this distinction between agents, I was mainly interested in studying the impact of introducing workers, who are discouraged from saving by rates of return that are lower than the rate of time preference. Evidently, this is not the case when the liquidity constraints are absent in the model and, thus, such claim made no economic sense in that context. However, by creating this distinction we are able to distinctly observe the effects of two different impacts of adding up liquidity constraints in the model of KM: on the one hand, liquidity constraints have a direct impact on the RBC statistics that comes from the equilibrium conditions of the model; on the other hand, liquidity constraints legitimate the claim that workers will not save in the steady state, by driving the return on both equity and money, to levels that are lower than the time preference rate. Indeed, it is this indirect effect that plays the most important role in

this model.

Table 5: Business Cycle Statistics for KM model with stochastic A_t :				
Variable	St Dev	Relative St Dev	Autocorrelation	Correlation with Y
Y	1.60	1.00	0.72	1.000
C	1.40	0.87	0.72	1.000
I	2.55	1.59	0.72	0.999
L	1.00	0.63	0.72	1.000
Y/L	0.60	0.38	0.72	1.000
w	0.60	0.38	0.72	1.000
r	0.08	0.05	0.72	0.990
A	0.94	0.59	0.72	0.997
q	0.39	0.24	0.72	0.992
p	1.25	0.78	0.72	1.000

If one compares the results in Table 4 with the ones in Table 5, one can see that the liquidity constraints *per se*, only emphasize slightly more the reduction in the volatility of investment and the increased variation of consumption. We can therefore conclude, that the main departure of these RBC statistics from the results produced by a standard Neoclassical one, derives from the fact that including financial constraints brings the return on equity and money lower than the subjective discount rate: those who do not face investment opportunities have no incentives to save, and this ultimately results in a major increase in the volatility of consumption at the expense of investment. Consequently, one can ascertain that the liquidity constraints decrease the volatility of investment in the model, not so much directly, through their impact on the optimal decision of entrepreneurs, but rather indirectly by inhibiting workers from saving.

It is now time to simulate the original KM model with both the technology

parameter A_t and the resaleability constraint φ_t defined as stochastic. The introduction of φ_t as a stochastic variable will provoke crucial changes to the statistics presented in Table 5. First of all, and most notoriously, the volatility of asset prices and investment suffers a remarkable increase. The standard deviation of q_t jumps from 0.39% to 1.97% which is almost in line with the 2.15% volatility Tobin's q observed for the U.S. economy and found by LS. I interpret the results for the volatility of p_t , by constructing an historical series for the 3-month Treasury Bill rate, using data from Table H.15 of the Federal Reserve Statistical Release. For the period of analysis between 1953 and 2000, I find that the quarterly volatility of the 3-month T-Bills rate is 2.80% relative to its steady state value. We can compare this value with the standard deviation coefficient for the Return On Money $r_t^M = \frac{p_{t+1} - p_t}{p_t}$, resulting from the model of 0.40%. We can see that, although the standard deviation of p_t is much more significant when KM is fed with liquidity shocks, the model is still not able to retrieve a variability in the return of money that is consistent with the data. In fact, this model still underestimates the variability of p_t , even though we can see a clear improvement in predicting the observed volatility, once φ_t is allowed to stochastically change over time. As a consequence of more volatile asset prices, Investment now fluctuates more than two times as much as output. This is still short when compared to the data, but constitutes a great improvement when compared with the results in Table 4. Although less impetuously, consumption also increases its variability from 1.40% to 1.49% and is now 90% as volatile as output. Unfortunately, it turns out that, adding the shock to the resaleability constraint further deteriorates the ability of this model to retrieve a consumption volatility that is in line with the U.S. data. All other aggregates and prices maintain their standard variations fairly at the same level.

Intuitively, persistence is not altered in this model, given that I have in-

roduced a shock that is basically as persistent as the technology shock. On the contrary, in what concerns comovement, some alterations are in place. Investment loses much of its contemporaneous correlation with output and is now fairly well predicted by the model. In fact the correlation coefficient for investment changes from broadly 1.00 in Table 4 to 0.75 in Table 5, which compares to the observed 0.80 as presented in Table 1. Interestingly, this is a much better result than the one obtained in the standard Neoclassical model simulation, presented in Table 2. As for consumption, there is a reduction in the contemporaneous correlation with output (from 1.00 to 0.94) that leaves this coefficient fairly similar to the one predicted by a standard Neoclassical model. Finally, also p and q become much less correlated with output. Recall from Table 5 that asset prices moved almost one to one with the product. Instead, in Table 6 it is clear that both q and p are now hardly procyclical, with correlation coefficients of just 0.201 and 0.105 respectively. All other variables maintain their comovement almost unchanged.

Table 6: Business Cycle Statistics for KM model with stochastic A_t and φ_t :				
Variable	St Dev	Relative St Dev	Autocorrelation	Correlation with Y
Y	1.61	1.00	0.73	1.000
C	1.49	0.93	0.73	0.942
I	3.40	2.12	0.72	0.745
L	1.00	0.63	0.73	1.000
Y/L	0.60	0.38	0.73	1.000
w	0.60	0.38	0.73	1.000
r	0.08	0.05	0.72	0.981
A	0.94	0.58	0.72	0.994
q	1.97	1.25	0.72	0.201
p	12.61	7.85	0.72	0.105

The introduction of shocks to the resaleability of equity defined as in KM brings about a fairly good prediction of the RBC statistics for the U.S. economy, even though investment is less volatile than desirable and consumption less smooth than appropriate. Moreover, another achievement of this model is that it justifies the volatility in asset prices as a normal feature of a monetary economy that is subject to liquidity shocks, even though asset prices still fluctuate more in the data than in the model. It is also interesting to see the decomposition of the variance, as presented in Table 7 in appendix: although φ_t only justifies a tiny fraction of the variation in output, labour, labour productivity, wages and gross profit rate, it explains 12% and 43% of the variances of consumption and investment, respectively, and essentially all the variation of asset prices p and q .

It is useful to recapitulate what has been done so far. I have started by replicating a standard Neoclassical model that included a stochastic shock to productivity and simulated it to obtain a benchmark for how well a model should behave, in predicting the RBC statistics for the U.S. economy. Once I have introduced the GHH preferences and the distinction between workers - who consumed all their income by definition - and capital owners, I have discovered that the same stochastic shock to productivity led to a great reduction in the ability of the model to predict the volatility of investment and consumption, verified in the data. Indeed, consumption gains too much impetus and varies almost as much as output, whereas investment sees its variability decrease from three times that of output to nearly the same level of variation. The next step was to include financial frictions. For that purpose I have introduced the liquidity constraints θ and φ set to be constant in the model. These justified the claim that, in equilibrium, workers would not want to save, given that the steady state rates of return on savings resulted lower than the time preference rate. Finally, I have taken the last step and allowed the resaleabil-

ity coefficient φ_t to vary stochastically. A reasonable calibration for the law of motion of φ_t recovered some of the investment's volatility (although not enough to approximate the one observed in the data and reproduced by the standard Neoclassical model) and set its contemporaneous correlation with output more or less in line with the data. Consumption volatility, instead, drifted further away from the observed 74%, to 93% as variable as output. At last, asset prices volatility jumped to much greater levels - much closer to the ones observed in the data -, denouncing the ability of this model to explain the paradox of excess asset price variability.

A crucial characteristic of this simulation is worth emphasizing as a final result. Although it is clear that, in the steady state, the rate of return on both assets is lower than the time preference rate, it is not certain that, when one feeds this economy with reasonably calibrated shocks to liquidity and technology, these rates of return will not respond strongly enough to exceed the subjective discount rate. In fact, this would contradict Claim 1, according to which workers want to consume all their labour income. It turns out that this claim is frequently violated in this simulation, as I am about to check. In Figure 1a and Figure 1b I present a simulated time series with 1000 periods for the Return on Money (ROM) and Return on Equity (ROE) - as perceived by a saver - against the Time Preference Rate (TPR). The series for the model where only A_t is stochastic are represented in dashed lines, whereas the series for the original KM model with φ_t stochastic are represented in solid lines.

It is clear-cut, for the time periods represented, that the return on money is smooth enough not to beat the time preference rate, even when liquidity shocks are fed into the model. However, in Figure 1b one can see that the ROE frequently exceeds the TPR in the original KM model. In fact, only in the case when φ is defined as a fixed variable, will this simulation produce a ROE that does not surpass the TPR. We can therefore conclude that Claim

1 is violated very frequently by this model, and that although the claim that workers consume all their income is valid for the steady state of this economy, the same is not true when one feeds this model with stochastic shocks that are calibrated for the U.S. economy.

Figure 1a: ROM vs TPR:

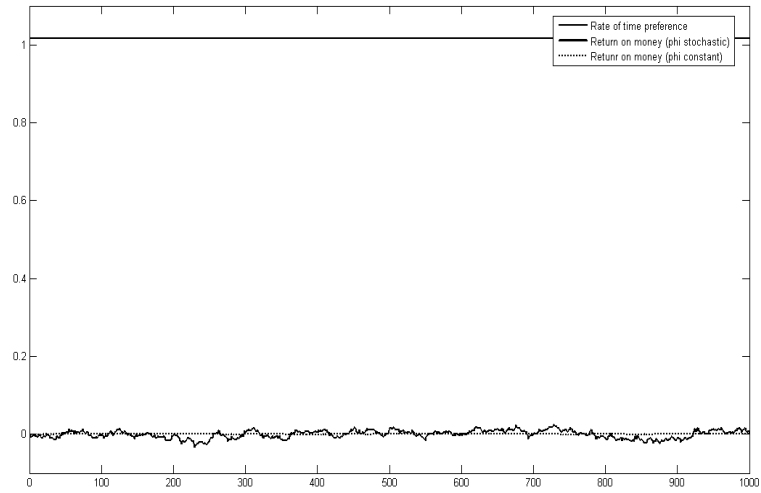
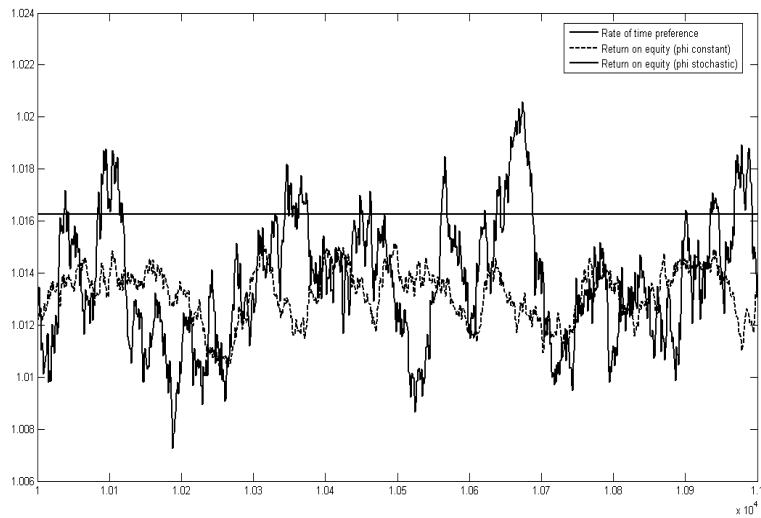


Figure 1b: ROE vs TPR:



Recall that Claim 1 is the main cause for the reduced volatility of investment and increased variability of consumption, and the sole explanation, put forward by the authors, for the reduced rate of participation in asset-markets. The violation of this claim, however, raises some doubts on the robustness of the results obtained in the simulation performed in this section and should therefore be taken care of.

Allowing workers to occasionally save will allow us to avoid this violation. However, such alteration is beyond the scope of this article. I should, nonetheless, emphasize the need to solve for this structural problem, in future research regarding the RBC properties of KM, if we want to seriously consider this model for policy analysis purposes.

7 Conclusion:

In the previous sections of this article I have exhaustively analyzed the RBC performance of the KM model. Broadly speaking, this new framework for studying money and liquidity, in a DSGE environment, proves to be very well equipped to replicate some remarkable features of the U.S. real business cycle statistics.

From the simulation results obtained, we can conclude that KM, not only maintains some crucial results from the simulation of a standard Neoclassical model (despite the decrease in the volatility of investment and the increased variability of consumption), but also provides some very positive contributes to improving the RBC performance of Neoclassical models. In fact, the simulation results for KM bring about a better prediction for the volatility of hours, real wages and labour productivity, together with a more precise estimation of the contemporaneous correlation coefficient between investment and output, when compared to the benchmark KR model. Moreover, and most striking of all, the simulation of KM retrieves a volatility of asset prices that

constitutes a fairly good prediction of the observed variation for the post-War U.S. economy.

Scrutinizing the impact of each new feature in KM, as compared to a standard Neoclassical model, also proved fruitful in understanding the structural causes for the previously referred breakthrough results. Indeed the following conclusions were taken:

- GHH preferences, alone, provide better simulation results for the volatility of hours, real wage rate and labour productivity, despite the deterioration in the comovement coefficients of most variables.
- Including liquidity constraints deteriorates the capacity of the model to produce the observed volatility of consumption and investment. The reason for this hinges, above all, on the fact that Claim 1, according to which workers prefer not to save, is legitimated, rather than on the impact caused by φ and θ on the equilibrium conditions of entrepreneurs.
- Allowing for shocks to the resaleability of equity (φ_t) is essential to bring back part of the observed volatility of investment (as well as its comovement coefficient) and most of the variation of asset prices.

Despite these positive conclusions, one note of caution should be made: the simulation results obtained rely on the assumption that workers never want to save, given the low steady state saving rates. However, as I have verified in the Section 6, this claim is very often broken, when shocks to A_t and φ_t , calibrated for the U.S. economy, are fed into the model. My prediction is that if one allows for workers to save, this model will recuperate the ability to produce the observed volatility of consumption and investment and also further intensify the volatility of asset prices, which, I recall, is still underestimated by KM.

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A Appendix

Table 7: Variance Decomposition of the KM model with A_t and φ_t stochastic

Variable	From σ_A (in %)	From σ_φ (in %)
Y	99.47	0.53
C	87.79	12.21
I	56.29	43.71
L	99.47	0.53
Y/L	99.47	0.53
w	99.47	0.53
r	99.71	0.29
q	3.94	96.06
p	0.99	99.01

Figures:

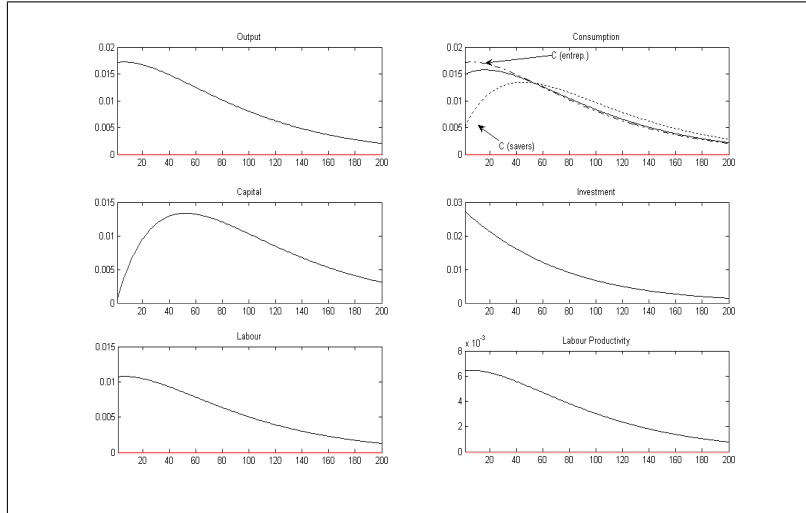


Figure 2a: Orthogonalized shock to A_t - Aggregates

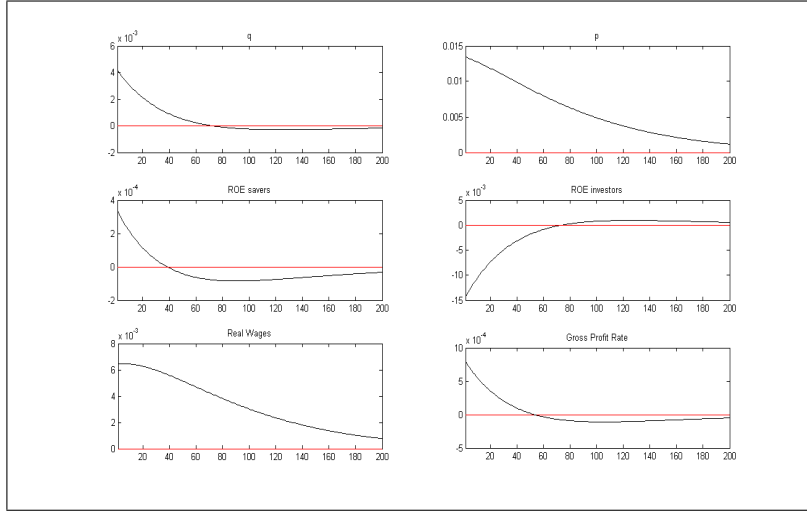


Figure 2b: Orthogonalized shock to A_t - Prices

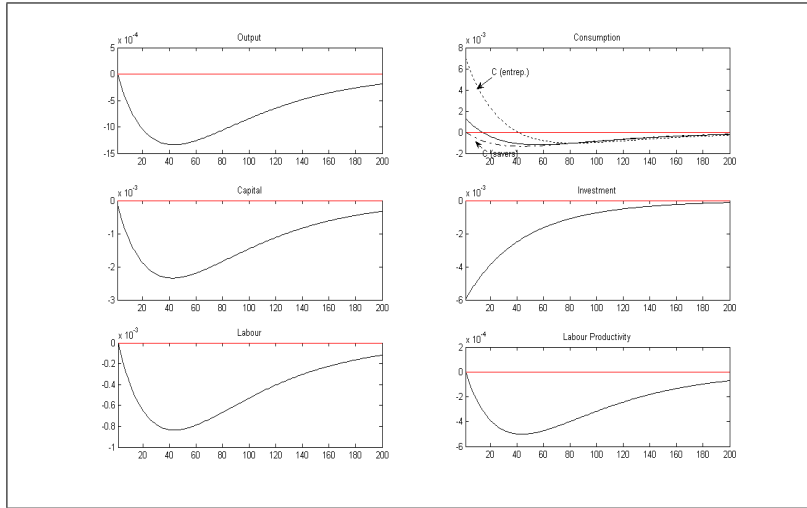


Figure 3a: 1pp orthogonalized shock to φ_t . - Aggregates:

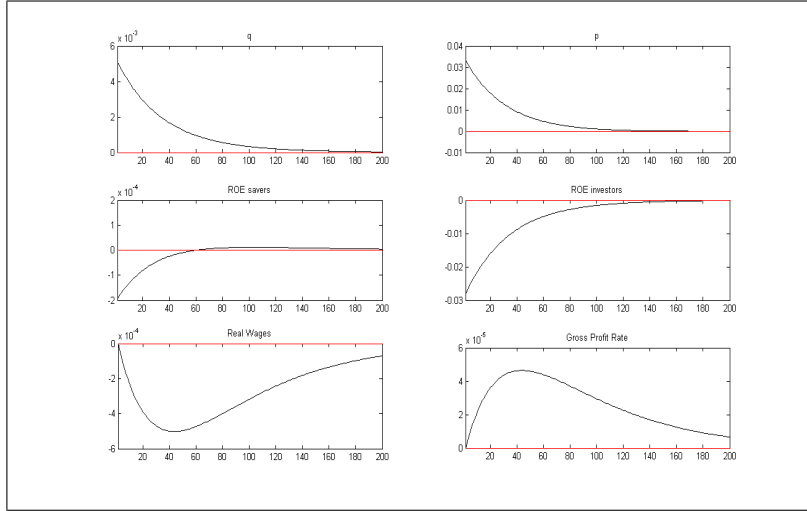


Figure 3b: 1pp orthogonalized shock to φ_t . - Prices

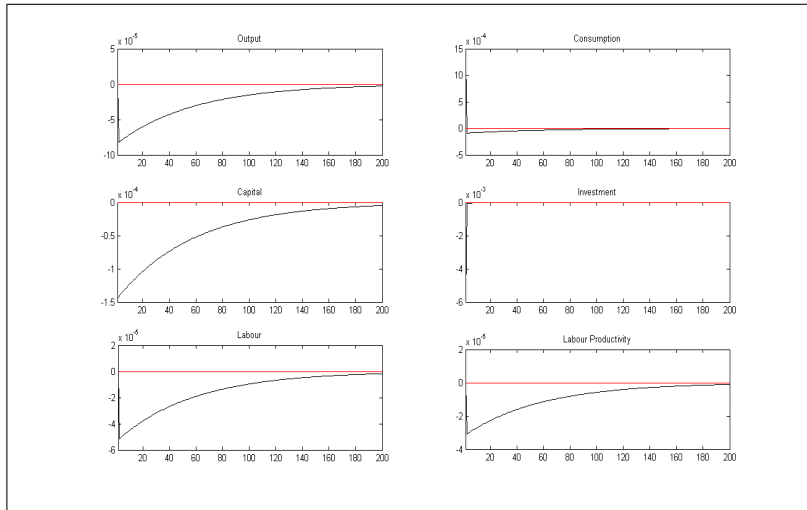


Figure 3c: 1pp orthogonalized transitory shock to φ_t . - Aggregates

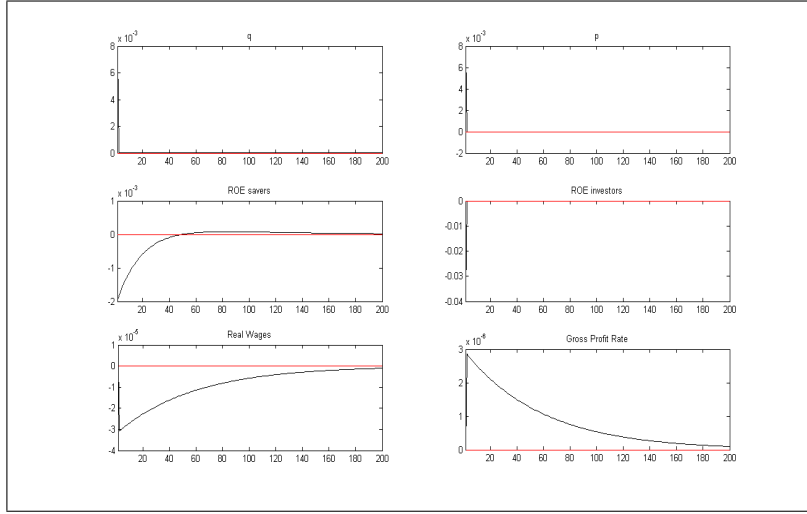


Figure 3d: 1pp orthogonalized transitory shock to φ_t . - Prices

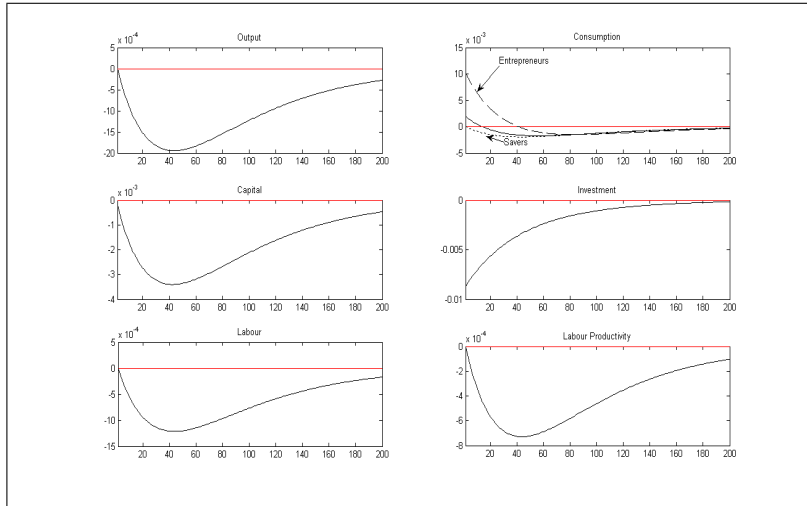


Figure 4a: 1pp orthogonalized shock to θ_t . - Aggregates

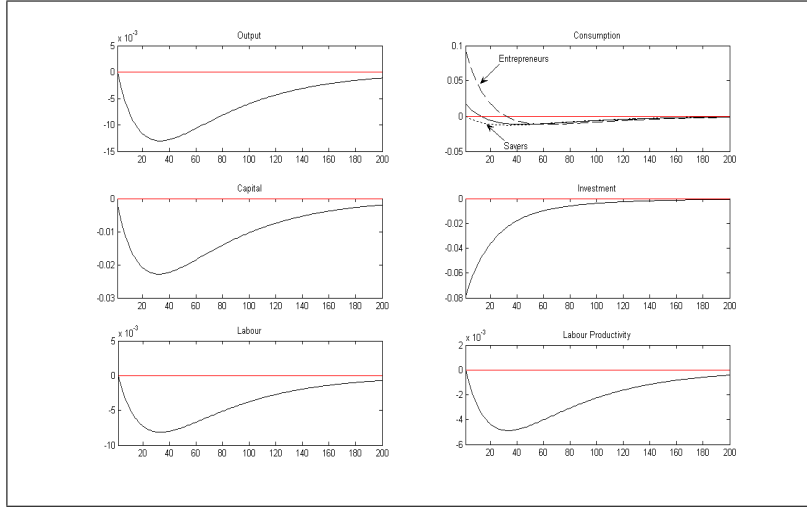


Figure 5a: 1pp orthogonalized shock to π . - Aggregates

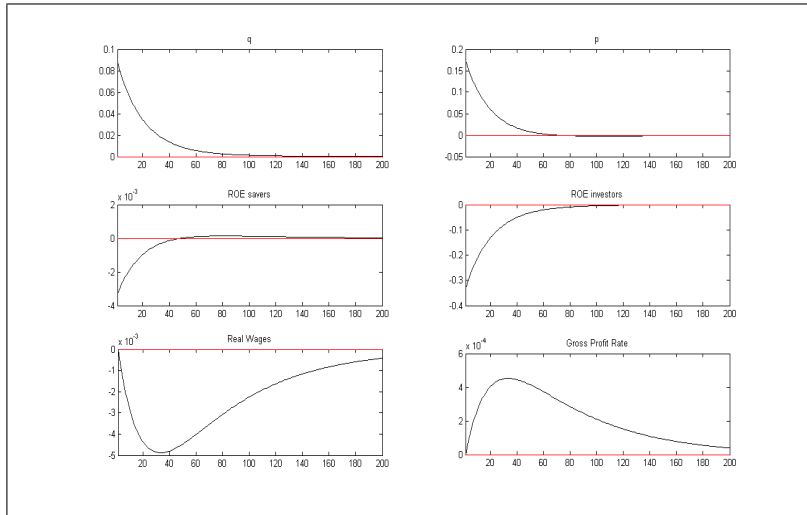


Figure 5b: 1pp orthogonalized shock to π - Prices